WHAT STARTS HERE CHANGES THE WORLD
THE UNIVERSITY OF TEXAS AT AUSTIN

Data Driven Application System For Laser Treatment of Cancer

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Team

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Outline

- Main Ideas

- Optimization Problems

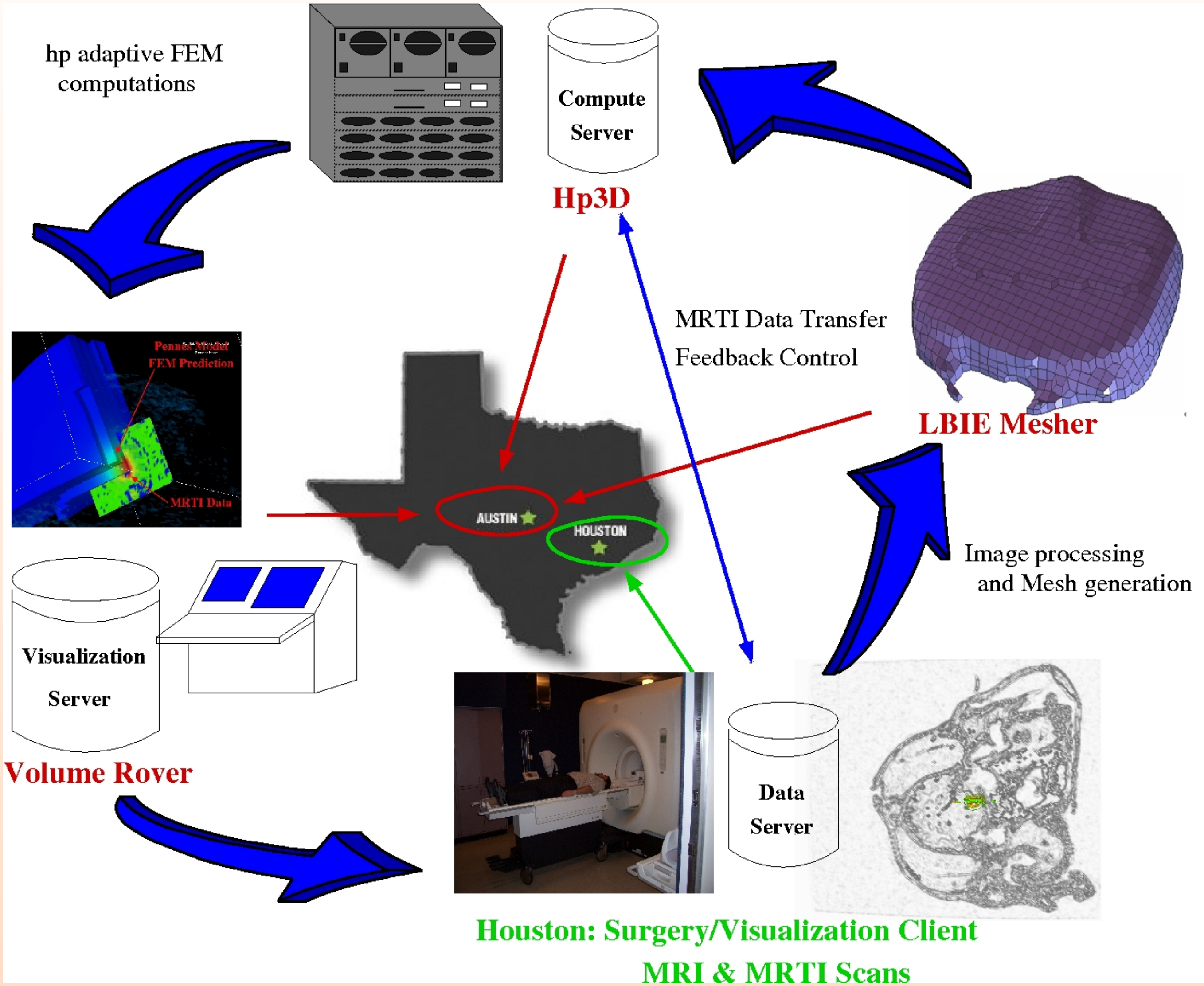
- Registration

- Execution Results

Main Ideas

- Computer guided laser treatment as a minimally invasive alternative to standard treatment of cancer
- Simple Idea: Subject all cells, including cancer cells, to temperatures outside normothermia range may damage and destroy cells
  - ▹ *Subject cells to hyperthermia/ablation temperature ranges*
  - ▹ *Heat source provided by diffusing interstitial laser fiber or collimated external source*
- Real-Time Thermal Imaging provides guidance to Real-Time computational prediction
- Target disease: tissues with a well-defined tumor

CyberInfrastructure



Bioheat Transfer Model

The Non-Linear Pennes Model

$$\rho c_p \frac{\partial u}{\partial t} - \nabla \cdot (k(u)\nabla u) + \omega(u) c_{blood}(u - u_a)$$
$$= Q_{laser}(\mathbf{x}, t) \ \textbf{in} \ \Omega \subset \mathbb{R}^3$$

$$-k(u)\nabla u \cdot \mathbf{n} = h(u - u_\infty) \qquad \textbf{on} \ \partial\Omega$$

$$u(\mathbf{x}, 0) = u^0 \qquad \textbf{in} \ \Omega$$

$$Q_{laser}(\mathbf{x}) = 3P\mu_a \frac{\exp(-\mu_s \|\mathbf{x} - \mathbf{x}_0\|)}{4\pi\|\mathbf{x} - \mathbf{x}_0\|}$$

Optimizations

The three main problems in which we are interested in is the real-time solution of the following problems

- **calibration of the model coefficients**

- **optimal control of the laser**

- **goal oriented error estimation**

Optimization Framework

$$\boxed{\begin{aligned}&\textbf{Find } q^* \in \mathbb{P} \textbf{ s.t.}\\ &\qquad Q(u(q^*), q^*) = \inf_{q \in \mathbb{P}} Q(u(q), q)\end{aligned}}$$

$u \in \mathcal{V}$, **determined from variational PDE**

$$C(u, q; v) = 0 \qquad \forall v \in \mathcal{V}$$

$\mathcal{V}$ **is the appropriately chosen Hilbert space, and** q **is a parameter in the control space** $\mathbb{P}$

$$\mathbb{P} \equiv \{q \in \mathcal{P} : \quad \exists! u \quad \textbf{s.t. } C(u, q; v) = 0 \qquad \forall v \in \mathcal{V}\}$$

Adjoint Method

Adjoint Problem

$$-\rho c_p \frac{\partial p}{\partial t} - \nabla \cdot (k(u)\nabla p) + \frac{\partial k}{\partial u}(u,\beta)\nabla u \cdot \nabla p$$
$$+ \omega(u,\beta)\, p + \frac{\partial \omega}{\partial u}(u,\beta)\, p\, (u - u_a)$$
$$= (u - \phi(\mathbf{x})) \qquad \textbf{in } \Omega$$

$$-k(u)\nabla p \cdot \mathbf{n} = h\, p \qquad \textbf{on } \partial\Omega$$
$$p(\mathbf{x},\tau) = 0 \qquad \textbf{in } \Omega$$

Adjoint Method

Gradient Computation

$$
\begin{bmatrix}
\int_0^{\tau}\int_{\Omega}\frac{\partial k}{\partial k_0}(u,\beta)\nabla u\cdot\nabla p\;dxdt \\
\int_0^{\tau}\int_{\Omega}\frac{\partial k}{\partial k_1}(u,\beta)\nabla u\cdot\nabla p\;dxdt \\
\int_0^{\tau}\int_{\Omega}\frac{\partial k}{\partial k_2}(u,\beta)\nabla u\cdot\nabla p\;dxdt \\
\int_0^{\tau}\int_{\Omega}\frac{\partial k}{\partial k_3}(u,\beta)\nabla u\cdot\nabla p\;dxdt \\
\int_0^{\tau}\int_{\Omega}\frac{\partial \omega}{\partial \omega_0}(u,\beta)(u-u_a)p\;dxdt \\
\int_0^{\tau}\int_{\Omega}\frac{\partial \omega}{\partial \omega_1}(u,\beta)(u-u_a)p\;dxdt \\
\int_0^{\tau}\int_{\Omega}\frac{\partial \omega}{\partial \omega_2}(u,\beta)(u-u_a)p\;dxdt \\
\int_0^{\tau}\int_{\Omega}\frac{\partial \omega}{\partial \omega_3}(u,\beta)(u-u_a)p\;dxdt
\end{bmatrix}
$$

Adjoint Method

Gradient Computation

$$\begin{bmatrix} \int_0^{\tau} \int_{\Omega} \frac{\partial Q_{laser}}{\partial P} p \, dx dt \\ \int_0^{\tau} \int_{\Omega} \frac{\partial Q_{laser}}{\partial \mu_a} p \, dx dt \\ \int_0^{\tau} \int_{\Omega} \frac{\partial Q_{laser}}{\partial \mu_s} p \, dx dt \\ \int_0^{\tau} \int_{\Omega} \frac{\partial Q_{laser}}{\partial x_0} p \, dx dt \\ \int_0^{\tau} \int_{\Omega} \frac{\partial Q_{laser}}{\partial y_0} p \, dx dt \\ \int_0^{\tau} \int_{\Omega} \frac{\partial Q_{laser}}{\partial z_0} p \, dx dt \end{bmatrix}$$

Adjoint Method

Goal Oriented Error Estimation

$\mathcal{O}(p)$ for state variable, $\mathcal{O}(p+1)$ for adjoint variable

$$\int_0^\tau \int_\Omega \left[\rho c_p \frac{\partial u}{\partial t} p + k(u,\beta) \nabla u \cdot \nabla p + \omega(u,\beta) c_{blood}(u - u_a)\, p \right] dx dt$$
$$+ \int_0^\tau \int_{\partial\Omega_C} h(u - u_\infty)\, p\, dA dt - \int_0^\tau \int_\Omega Q_{laser}(\mathbf{x},\eta) p\, dx dt$$

Registration

- Currently have capabilities for rigid body registration

- Using ITK for Registration.

- www.itk.org

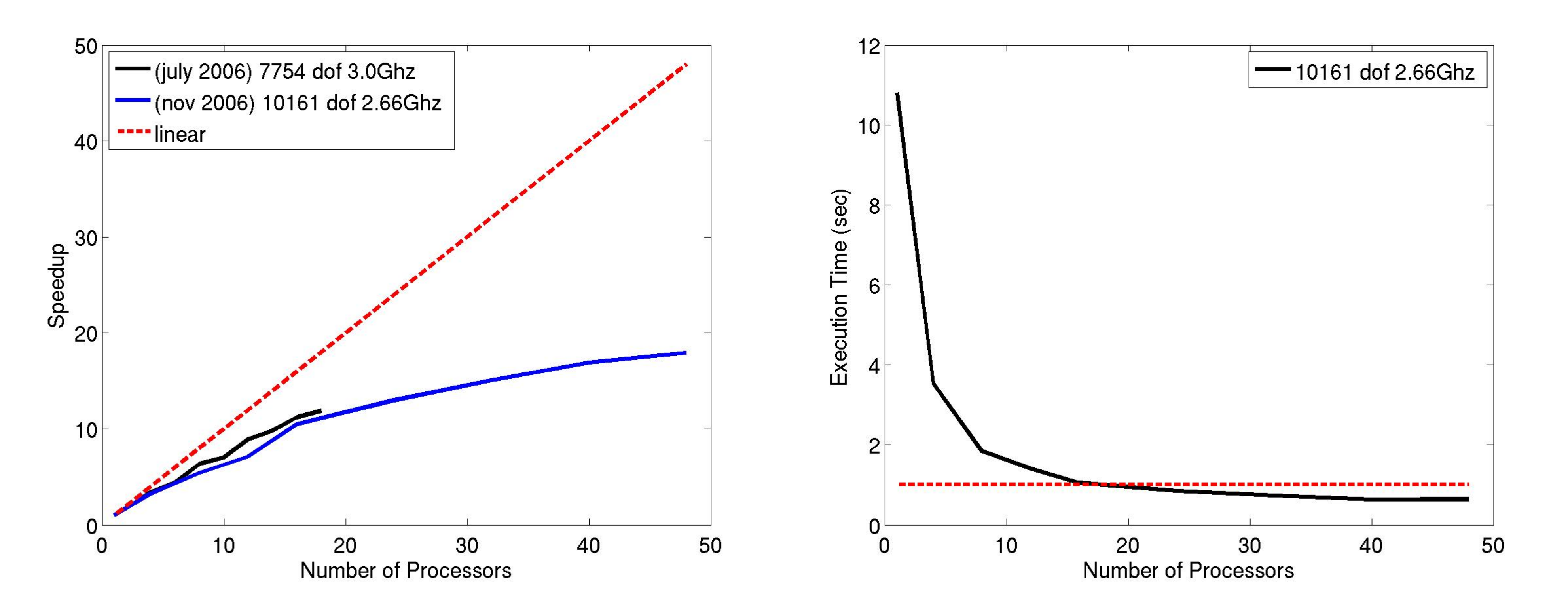
Alternative objective functions

$$Q(u(\beta),\beta) = \begin{cases} \frac{1}{2}\|u(\mathbf{x},t) - u_{ideal}(\mathbf{x},t)\|^2_{L^2\left([0,\tau];L^2(\Omega)\right)} & \textbf{Temp} \\ \frac{1}{2}\|D(u) - D_{ideal}(\mathbf{x})\|^2_{L^2(\Omega)} & \textbf{Damage} \\ \frac{1}{2}\|H(u,t) - H_{ideal}(\mathbf{x},t)\|^2_{L^2\left([0,\tau];L^2(\Omega)\right)} & \textbf{HSP} \end{cases}$$

- **Colleagues in BME developing empirical models of HSP/Damage from in-vitro cellular data**

- **HSP/Damage-Based optimizations to provide ideal HSP/Damage field**

Code Execution



Current Capabilities: 40-50sec prediction

or ≈5 gradient computations in 10sec

Data Transfer

- **Houston to Austin**
  - *34 anatomical images 131072 bytes each*
  - *121*5 temp images 65535 bytes each*
  - *avg bandwidth was .2MB/s and the file size of one time instance is .32MB*

- **Lonestar to Maverick**
  - *120 files sets use for vis of MRTI w/ fem.*
  - *5.5 MB FEM Mesh 2.6 MB MRTI Vis*
  - *avg bandwidth: 1.7MB/s (WORK FILE system)*
  - *avg bandwidth: 3.4MB/s (/tmp on the comp node)*

Concluding Comments

The possibility of reducing death due to cancer or enhancing the quality of life of patients may represent one of the great triumphs of Finite Element Methods and Computational Engineering

Questions

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