Laser Source Term

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Source Term for DDDAS

$$c\rho \frac{\partial T(\mathbf{x},t)}{\partial t} = \nabla\left(k\nabla T(\mathbf{x},t)\right) + \omega_b c_b \left(T(\mathbf{x},t) - T_a\right) + Q\left(\|\mathbf{x} - \mathbf{x_0}\|\right)$$

Lasers

There are two types of light

- Collimated Light - light whose rays are parallel
- Diffuse Light - light reflecting equally in all directions

Laser light is collimated light, and is such that all rays are of the same wavelength, λ

Energy from Light

We know from the First Law of Thermodynamics that

$$dE = dW + dQ$$

What energy does light have?

$$E = h\nu$$

where h is Plank's constant and ν is the wave frequency.

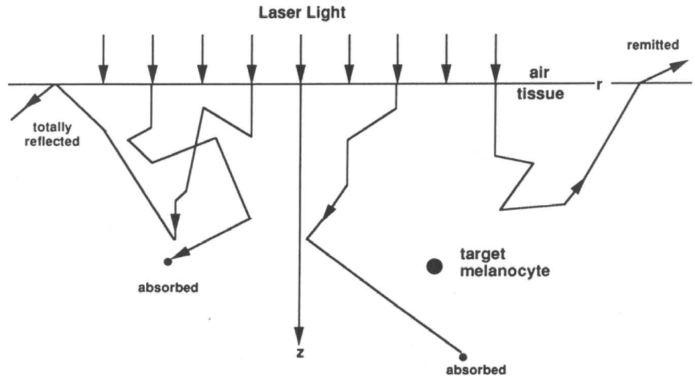
Now

$$\nu = \frac{c_m}{\lambda}$$

where c_m is the speed of light in whatever medium it is in, and λ is the wavelength. So, we have

$$E = h\frac{c}{\lambda}$$

Light enters tissue and takes various paths



Various particles in tissue will absorb the light or scatter the light.

Image taken from *Optical-Thermal Response of Laser Irradiated Tissue* A.J. Welch, et. al.

Energy is transferred when a light particle, or a photon, is absorbed into the medium.

Since these photons are not deforming the medium and do not have weight, there is no kinetic energy transfer, thus it must all be heating.

$$dE = dW + dQ = 0 + dQ$$

$Q\left(\|\mathbf{x}-\mathbf{x_0}\|\right)\ \left[\frac{\mathrm{W}}{\mathrm{cm}^3}\right]$ is meant to tell us how much heating per unit volume is happening. But since we are using light, this is equivalent to giving the rate of absorbed laser energy per unit volume.

Definitions

Optical Properties

- Absoprtion Coefficient μ_a $\left[\frac{1}{\mathrm{cm}}\right]$:
  probability of an absorption event per infinitesimal path length Δx is $\mu_a \Delta x$
- Scattering Coefficient μ_s $\left[\frac{1}{\mathrm{cm}}\right]$:
  probability of a scattering event per infinitesimal path length Δx is $\mu_s \Delta x$
- Attenuation Coefficient $\mu_t = \mu_a + \mu_s$ $\left[\frac{1}{\mathrm{cm}}\right]$:
  probability of an interaction per infinitesimal path length Δx is $\mu_t \Delta x$

We are currently using

- $\mu_a = .046$
- $\mu_s = 14.74$
- $\mu_t = 14.786$

all in $\left[\frac{1}{\mathrm{cm}}\right]$

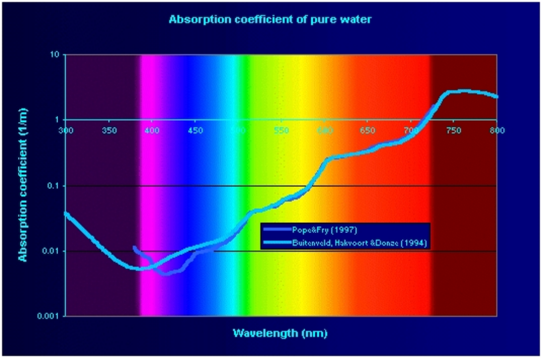


Image taken from www.deepocean.net/deepocean/science07.php

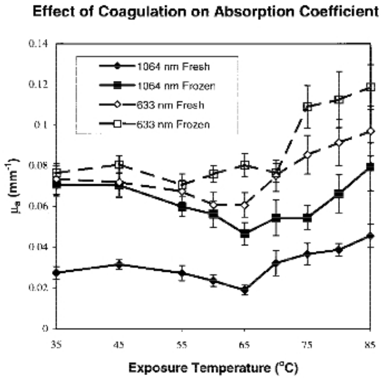
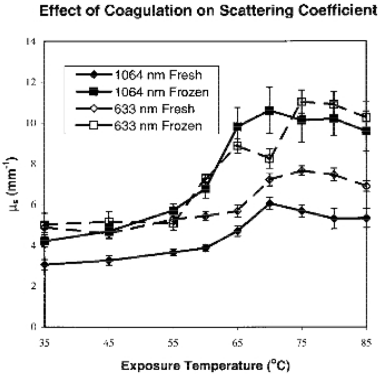


Image taken from *Measurement of Thermal Effects of the Optical Properties of Prostate Tissue at Wavelengths of 1064 and 633 nm* Nau, William et. al



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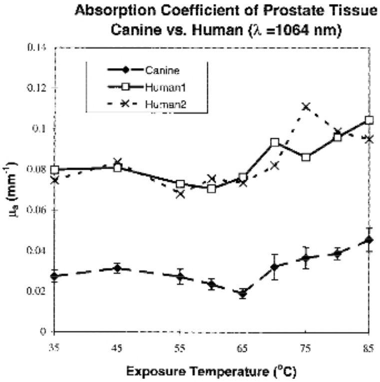

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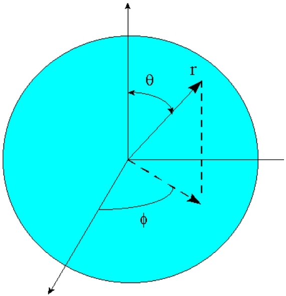
Optical Properties

- Phase Scattering Function $p(\hat{\mathbf{s}}, \hat{\mathbf{s}}')$: probability of a photon scattering into the $\hat{\mathbf{s}}$ direction from the $\hat{\mathbf{s}}'$ direction.

Note: We assume that $p(\hat{\mathbf{s}}, \hat{\mathbf{s}}') = p(\cos\theta, \varphi) = p(\cos\theta)\, p(\varphi)$, where θ is the angle off of the z-axis and φ is the angle off of the x-axis.



Optical Properties

- Anisotropic Factor g: the expected value of the cosine of the scattering angle θ

$$g = \int_{4\pi} p\left(\hat{\mathbf{s}}, \hat{\mathbf{s}}'\right)\left(\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}'\right)\ d\omega$$

Note: g ranges from -1 to 1, with $g = 0$ meaning that the scattering it isotropic. g usually is between .7 and .99 for tissues when the wavelength is in the visible and near-infrared wavelengths.

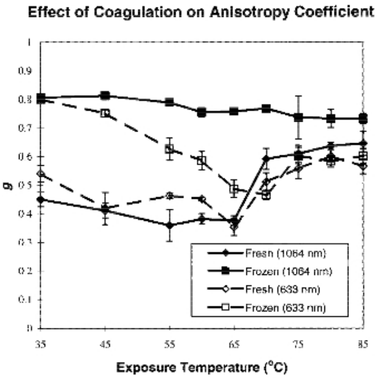


Image taken from *Measurement of Thermal Effects of the Optical Properteis of Prostate Tissue at Wavelengths of 1064 and 633 nm*

Optical Properties

- $N(\mathbf{r}, \hat{\mathbf{s}})$: the number of photons at a given point $\mathbf{r}$ moving in the direction of unit vector $\hat{\mathbf{s}}$

Note: If dA is an infinitesimal area perpendicular to $\hat{\mathbf{s}}$ located at $\mathbf{r}$, $d\omega$ is an infinitesimal solid angle about $\hat{\mathbf{s}}$, c_t is the photon velocity, and $h\nu$ (h being Plank's constant, and $\nu = c/\lambda$) is the energy per photon

$$N(\mathbf{r}, \hat{\mathbf{s}})\, dA d\omega c_t h\nu$$

yields the energy of the photons that propogate per second through dA at $\mathbf{r}$ within $d\omega$ in the direction $\hat{\mathbf{s}}$

Optical Properties

- Irrandiance $E(\mathbf{r}, \hat{\mathbf{s}})$ $\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$: rate of energy delivery per second per unit area of the tissue surface

Optical Parameters

- $L(\mathbf{r}, \hat{\mathbf{s}})$ $\left[\frac{\mathrm{W}}{\mathrm{sr \cdot m^2}}\right]$: Radiance gives the propagation of photon power.

$$L(\mathbf{r}, \hat{\mathbf{s}}) = N(\mathbf{r}, \hat{\mathbf{s}})\, h\nu c$$

Note: If $dP(\mathbf{r}, \hat{\mathbf{s}})$ is the power flowing through area dA at $\mathbf{r}$ in direction of $\hat{\mathbf{s}}$ within $d\omega$, then

$$dP(\mathbf{r}, \hat{\mathbf{s}}) = L(\mathbf{r}, \hat{\mathbf{s}})\, dA d\omega$$

Optical Properties

- Fluence Rate $\Phi(\mathbf{r})$ $\left[\frac{\mathrm{W}}{\mathrm{cm}^2}\right]$: the radiant power incident on a small sphere, divided by the cross-sectional area of that sphere

$$\Phi(\mathbf{r}) = \int_{4\pi} L(\mathbf{r}, \hat{\mathbf{s}})\; d\omega$$

Note: This has more practical significance than the radiance alone.

Optical Properties

- Primary Light: Light that has not undergone any scattering events (Sometimes called Collimated Light)
- Scattered Light: Light that has undergone at least one scattering event

We often write

$$\Phi_t(\mathbf{r}) = \Phi_p(\mathbf{r}) + \Phi_s(\mathbf{r})$$

Where Φ_t = total fluence, Φ_p = primary fluence and Φ_s = scattered fluence.

Optical Properties

- Net Flux Vector $F(\mathbf{r})$ $\left[\frac{\mathrm{W}}{\mathrm{cm}^2}\right]$:

$$F(\mathbf{r}) = \int_{4\pi} L(\mathbf{r}, \hat{\mathbf{s}})\, \hat{\mathbf{s}}\; d\omega$$

Optical Properties
Of physical importance is

$$\begin{aligned} F(\mathbf{r}) \cdot \hat{\mathbf{n}} &= \int_{4\pi} L(\mathbf{r}, \hat{\mathbf{s}})(\hat{\mathbf{s}} \cdot \hat{\mathbf{n}}) \; d\omega \\ &= \int_{\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}>0} L(\mathbf{r}, \hat{\mathbf{s}})(\hat{\mathbf{s}} \cdot \hat{\mathbf{n}}) \; d\omega + \int_{\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}<0} L(\mathbf{r}, \hat{\mathbf{s}})(\hat{\mathbf{s}} \cdot -\hat{\mathbf{n}}) \; d\omega \\ &= \int_{\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}>0} L(\mathbf{r}, \hat{\mathbf{s}})(\hat{\mathbf{s}} \cdot \hat{\mathbf{n}}) \; d\omega - \int_{\hat{\mathbf{s}}\cdot\hat{\mathbf{n}}<0} L(\mathbf{r}, \hat{\mathbf{s}})(\hat{\mathbf{s}} \cdot \hat{\mathbf{n}}) \; d\omega \\ &= F_{n+}(\mathbf{r}) - F_{n-}(\mathbf{r}) \end{aligned}$$

If the radiance is purely scattered light then

$$L\left(\mathbf{r}, \hat{\mathbf{s}}\right) = L_0\left(\mathbf{r}\right)$$

Further,

$$F_{n+} + F_{n-} = 2\pi L_0 = 2\pi \frac{\Phi_s}{4\pi}$$

That is

$$\Phi_s = 2\left(F_{n+} + F_{n-}\right)$$

The Analytic Solution

Since Q is to give us the amount of absorbed laser energy per unit volume, we have

$$Q(\mathbf{r}) = \mu_a \Phi(\mathbf{r})$$

We are currently using

$$\Phi(\mathbf{r}) = 3P(t)\,\mu_{tr}\frac{e^{-\mu_{eff}\mathbf{r}}}{4\pi\mathbf{r}}$$

where $\mu_{eff}^2 = 3\mu_a(\mu_a + \mu_s(1-g))$ and $\mu_{tr} = (\mu_a + \mu_s(1-g))$.

Theorem:
In a spherically symmetric geometry, if the diffusion approximation holds and the fluence is assumed isotropic
we have the form of the fluence as

$$\Phi_t(r) = \frac{3P_0}{4\pi}\left[\mu_{tr}\frac{e^{-\mu_{eff} r}}{r} - \frac{2}{3}\frac{e^{-\mu_t r}}{r^2}\right]$$